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# Introduction

Toward greater focus and coherence

For example, the number 10 can be represented as  $10 = 1 + 9$ ,  $10 = 2 + 8$ ,  $10 = 3 + 7$ ,  $10 = 4 + 6$ ,  $10 = 5 + 5$ ,  $10 = 6 + 4$ ,  $10 = 7 + 3$ ,  $10 = 8 + 2$ , and  $10 = 9 + 1$ . This understanding of how one number relates to another is foundational for understanding addition and subtraction.

Understanding the relationship between addition and subtraction is also important. For example, if  $8 + 4 = 12$ , then  $12 - 8 = 4$  and  $12 - 4 = 8$ . This understanding is essential for solving word problems involving unknowns in all positions.

Understanding mathematics involves more than just knowing facts and procedures. It also involves understanding the concepts and relationships that underlie the mathematics. For example, understanding the concept of place value is essential for understanding addition and subtraction. Understanding the concept of multiplication is essential for understanding area and volume. Understanding the concept of division is essential for understanding fractions and decimals.

## Understanding mathematics

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# How to read the grade level standards

**Standards** 1. Use addition and subtraction within 100 to solve problems involving unknowns in all positions.

**Clusters** 1. Understand, interpret, and use the standard form of numbers to represent real-world quantities.

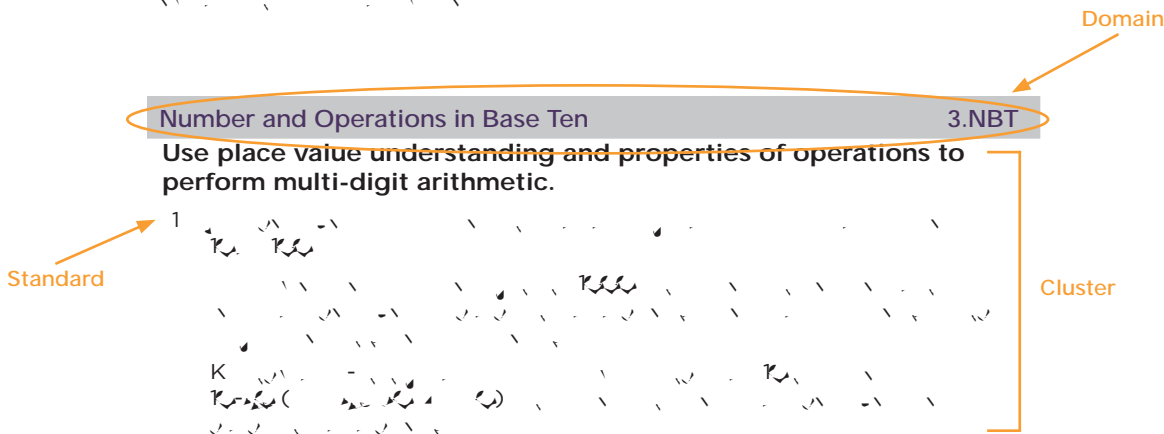
**Domains** 1. Understand, interpret, and use the standard form of numbers to represent real-world quantities.

**Number and Operations in Base Ten** **3.NBT**  
 Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Understand, interpret, and use the standard form of numbers to represent real-world quantities.

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### Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

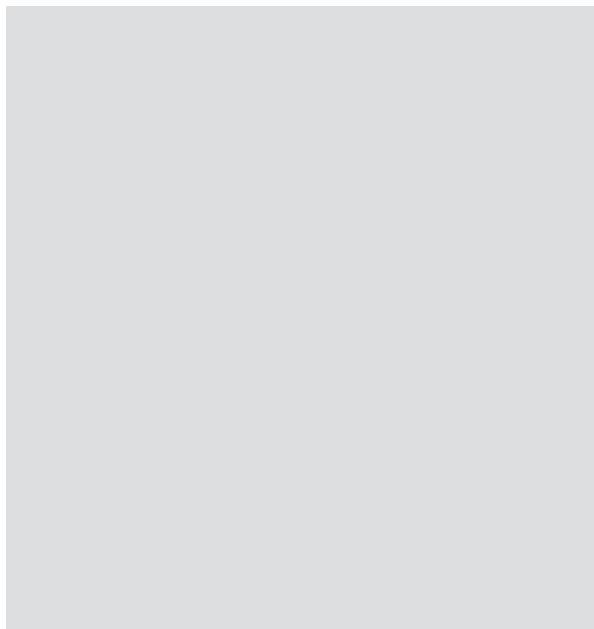
### Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

### Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

### Measurement and Data



## Counting and Cardinality

K.CC

**Know number names and the count sequence.**



**Count to tell the number of objects.**



**Compare numbers.**



## Operations and Algebraic Thinking

K.OA

**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**



## Number and Operations in Base Ten

K.NBT

Work with numbers 11–19 to gain foundations for place value.

1. Understand that the numbers 11–19 are composed of ten ones and one to nine more ones.

## Measurement and Data

K.MD

Describe and compare measurable attributes.

1. Describe objects in the world by their shapes, sizes, positions, and orientations.

Classify objects and count the number of objects in each category.

1. Classify two-dimensional shapes and three-dimensional objects by attributes.

## Geometry

K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Identify shapes and describe their defining attributes.

Analyze, compare, create, and compose shapes.

1. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, pentagons, hexagons, octagons) to create larger shapes.

# Mathematics | Grade 1

1.NB.A.1.1 (1)

# Grade 1 Overview

## Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

## Number and Operations in Base Ten

- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

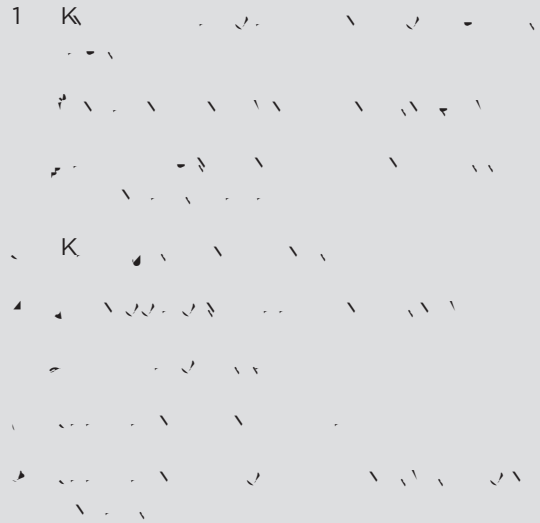
## Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

## Geometry

- Reason with shapes and their attributes.

## Mathematical Practices



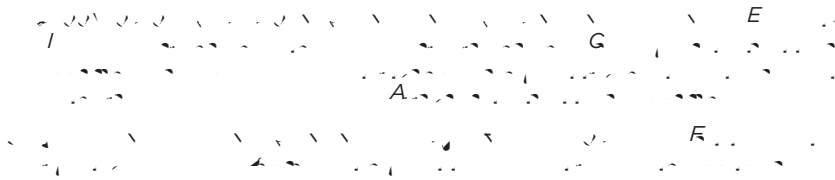
## Operations and Algebraic Thinking

## 1.OA

**Represent and solve problems involving addition and subtraction.**



**Understand and apply properties of operations and the relationship between addition and subtraction.**



**Add and subtract within 20.**



Use place value understanding and properties of operations to add and subtract.

Use place value understanding and properties of operations to add and subtract.

Use place value understanding and properties of operations to add and subtract.

Measurement and Data

1.MD

Measure lengths indirectly and by iterating length units.

1.MD.A.1 Measure length by iterating length units.



## Mathematics | Grade 2

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## Grade 2 Overview

### Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

### Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

### Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

### Geometry

- Reason with shapes and their attributes.

### Mathematical Practices

1. **K** **1** **2** **3** **4** **5** **6** **7** **8** **9** **10** **11** **12**
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## Measurement and Data

## 2.MD

## Measure and estimate lengths in standard units.

1. Measure the length of an object using standard units of length (e.g., inches, centimeters, feet). Add or subtract the lengths when joining two end-to-end objects of the same unit. Measure to the nearest whole unit.

## Relate addition and subtraction to length.

1. Measure the length of an object, and then use a length to measure another object. Use an object to measure the length of another object.

## Work with time and money.

1. Tell and write time in hours and half-hours using analog and digital clocks.
1. Know and use the relationships between dollars and cents, and between quarters and dimes, and between dimes and nickels, and between nickels and pennies.

## Represent and interpret data.

1. Represent data using a simple bar graph. Use a picture key to label the categories and count the number of objects in each category. Sort, add, subtract, and measure the length of the data.
1. Interpret data presented in a simple bar graph. Ask and answer questions about the data.

## Geometry

## 2.G

## Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of vertices or sides or a given angle. Classify shapes and figures by these attributes and create shapes from their attributes.
1. Partition a square into four smaller squares by a horizontal line and a vertical line. Partition a rectangle into two triangles by a diagonal line. Partition a circle into two halves by a diameter line. Partition a circle into three equal sectors by three radii meeting at the center. Partition a circle into four equal sectors by two diameters meeting at the center. Partition a circle into six equal sectors by three diameters meeting at the center.

K



## Grade 3 Overview

### Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

### Number and Operations in Base Ten

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

### Number and Operations—Fractions

- Develop understanding of fractions as numbers.

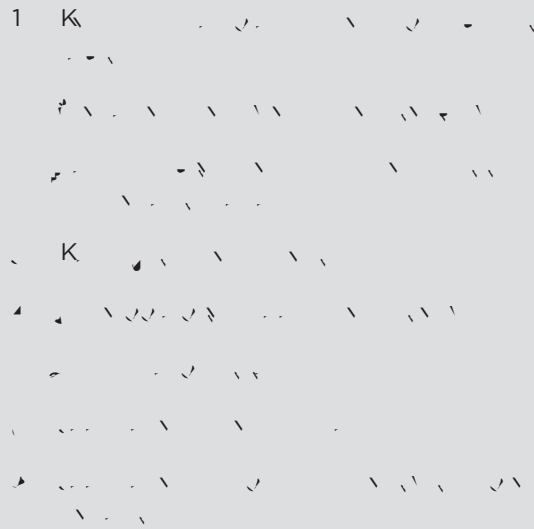
### Measurement and Data

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

### Geometry

- Reason with shapes and their attributes.

### Mathematical Practices









K-12

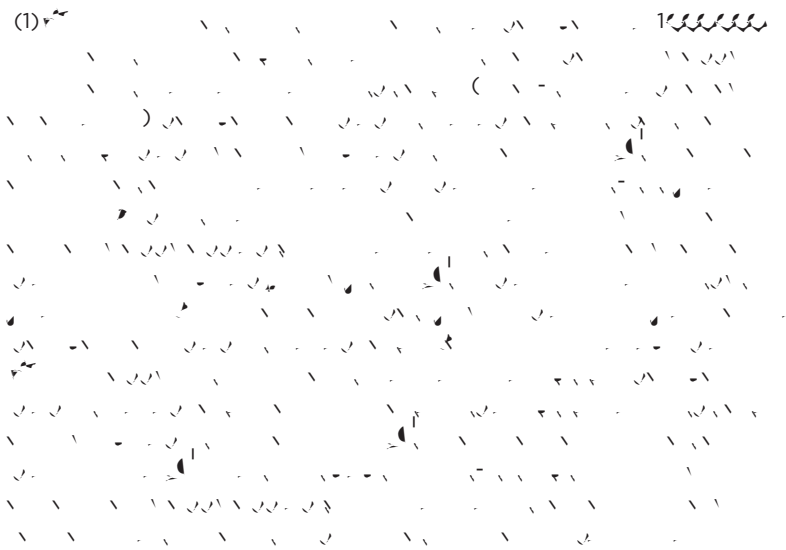
## Geometry

3.G

Reason with shapes and their attributes.



## Mathematics | Grade 4



## Grade 4 Overview

### Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

### Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

### Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

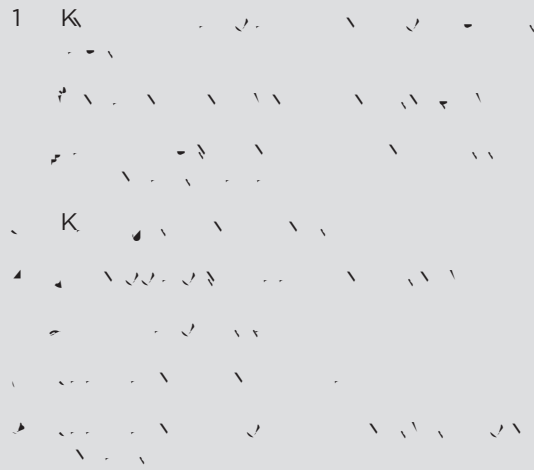
### Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

### Geometry

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

### Mathematical Practices





Number and Operations—Fractions<sup>3</sup>

## 4.NF

Extend understanding of fraction equivalence and ordering.

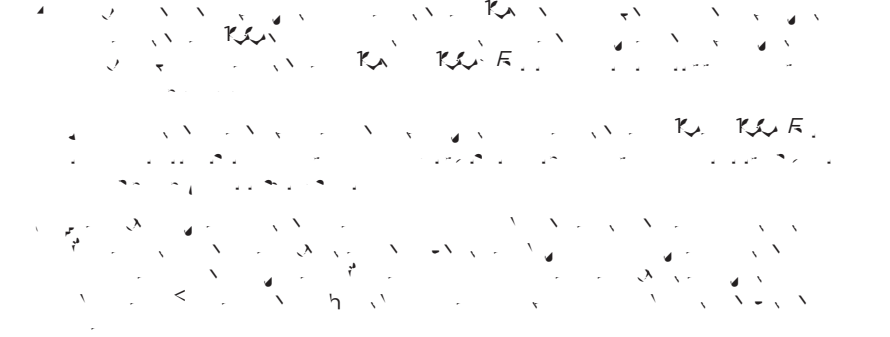
1. Compare two fractions having the same numerator or the same denominator by modeling the fractions on a number line or by using area models.



Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.



**Understand decimal notation for fractions, and compare decimal fractions.**



Measurement and Data

4.MD

**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**



**Represent and interpret data.**



**Geometric measurement: understand concepts of angle and measure angles.**







## Mathematics | Grade 5



## Operations and Algebraic Thinking

## Operations and Algebraic Thinking

5.OA

Write and interpret numerical expressions.



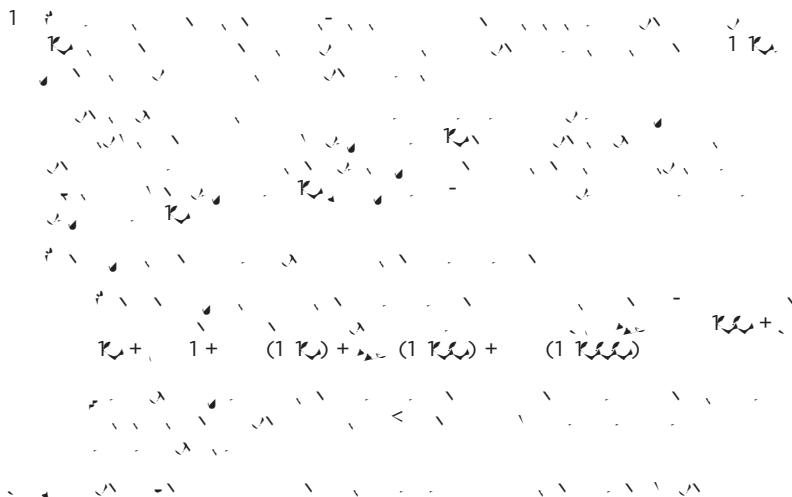
Analyze patterns and relationships.



## Number and Operations in Base Ten

5.NBT

Understand the place value system.



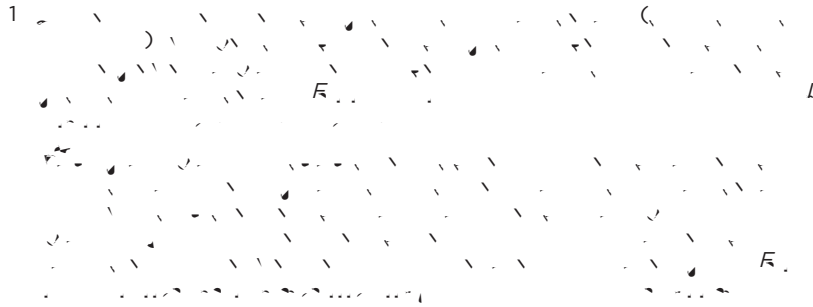
Perform operations with multi-digit whole numbers and with decimals to hundredths.



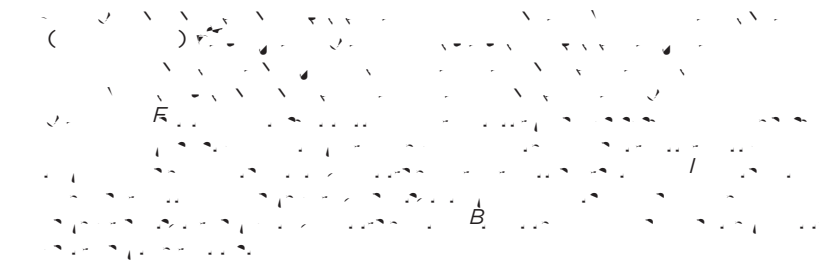
## Number and Operations—Fractions

## 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.



Apply and extend previous understandings of multiplication and division to multiply and divide fractions.



1 2 3 4 5 6 7 8 9 10 11 12



## Geometry

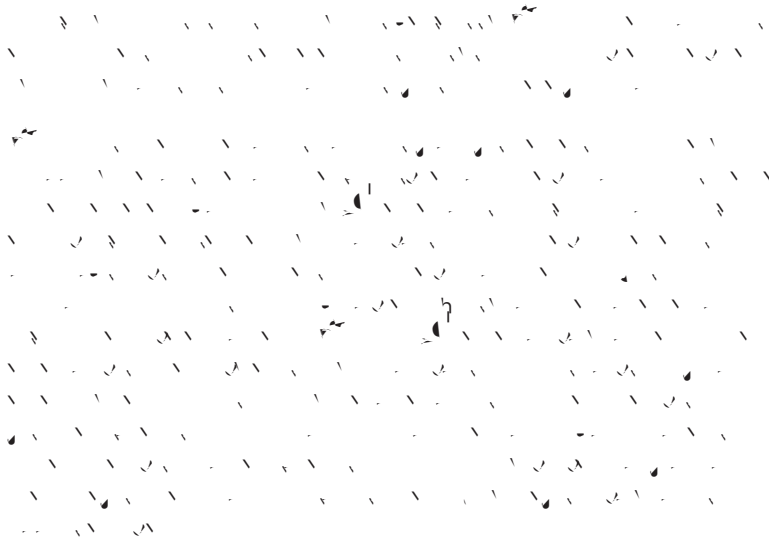
5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.



## Mathematics | Grade 6







## Grade 6 Overview

### Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

### The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

### Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

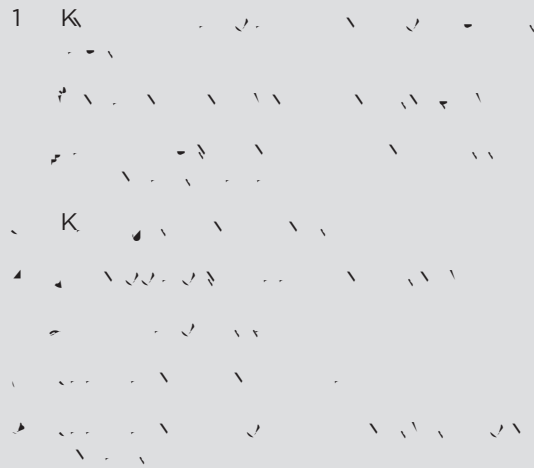
### Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

### Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

### Mathematical Practices



## Ratios and Proportional Relationships

6.RP

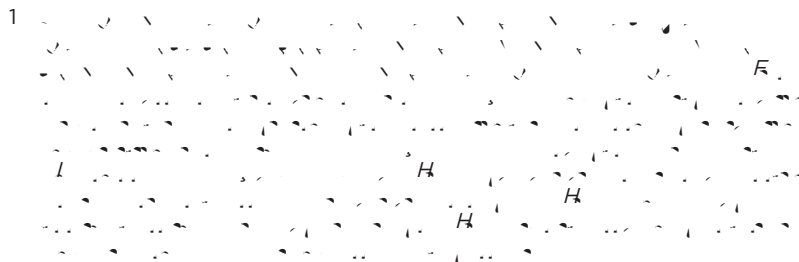
Understand ratio concepts and use ratio reasoning to solve problems.



## The Number System

6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

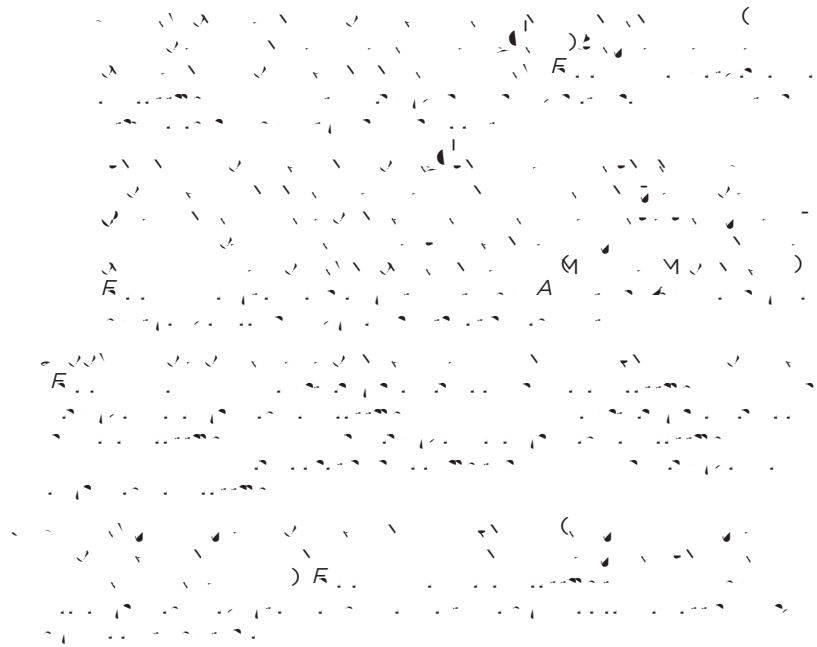


Compute fluently with multi-digit numbers and find common factors and multiples.



**Apply and extend previous understandings of numbers to the system of rational numbers.**





**Reason about and solve one-variable equations and inequalities.**





### Statistics and Probability

### 6.SP

#### Develop understanding of statistical variability.

1. Understand that statistics from a random sample can be used to answer questions about a population. For example, the number of hours per week that a sample of sixth graders spends watching television can be used to answer questions about the number of hours per week that all sixth graders spend watching television.

#### Summarize and describe distributions.

1. Represent data with plots on a number line, including dot plots, histograms, and box plots.
2. Summarize the distribution of data with statistical measures of center (mean, median, mode) and variability (range, interquartile range, standard deviation, and mean absolute deviation).

## Mathematics | Grade 7

(1)  $\frac{1}{2}$  of the number of students who are in the school choir is 15. How many students are in the school choir?  
 (2)  $\frac{1}{3}$  of the number of students who are in the school choir is 15. How many students are in the school choir?  
 (3)  $\frac{1}{4}$  of the number of students who are in the school choir is 15. How many students are in the school choir?  
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### Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

### The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

### Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

### Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

### Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

### Mathematical Practices



-11 ( )

## Ratios and Proportional Relationships

7.RP

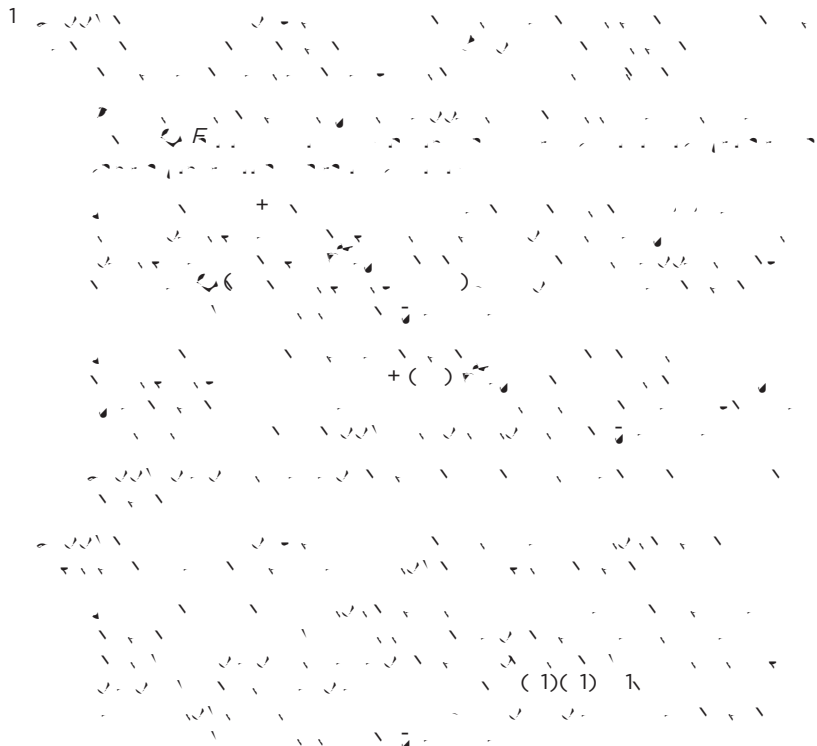
Analyze proportional relationships and use them to solve real-world and mathematical problems.



## The Number System

7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.









**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**



Statistics and Probability

7.SP

**Use random sampling to draw inferences about a population.**

1





## Grade 8 Overview

### The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

### Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

### Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

### Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

### Statistics and Probability

- Investigate patterns of association in bivariate data.

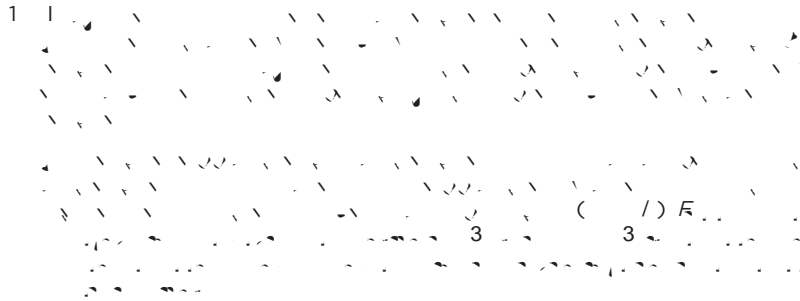
### Mathematical Practices



## The Number System

8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.



## Expressions and Equations

8.EE

Work with radicals and integer exponents.



Understand the connections between proportional relationships, lines, and linear equations.



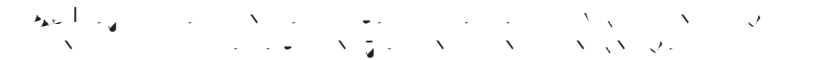




**Understand and apply the Pythagorean Theorem.**



**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**



Statistics and Probability

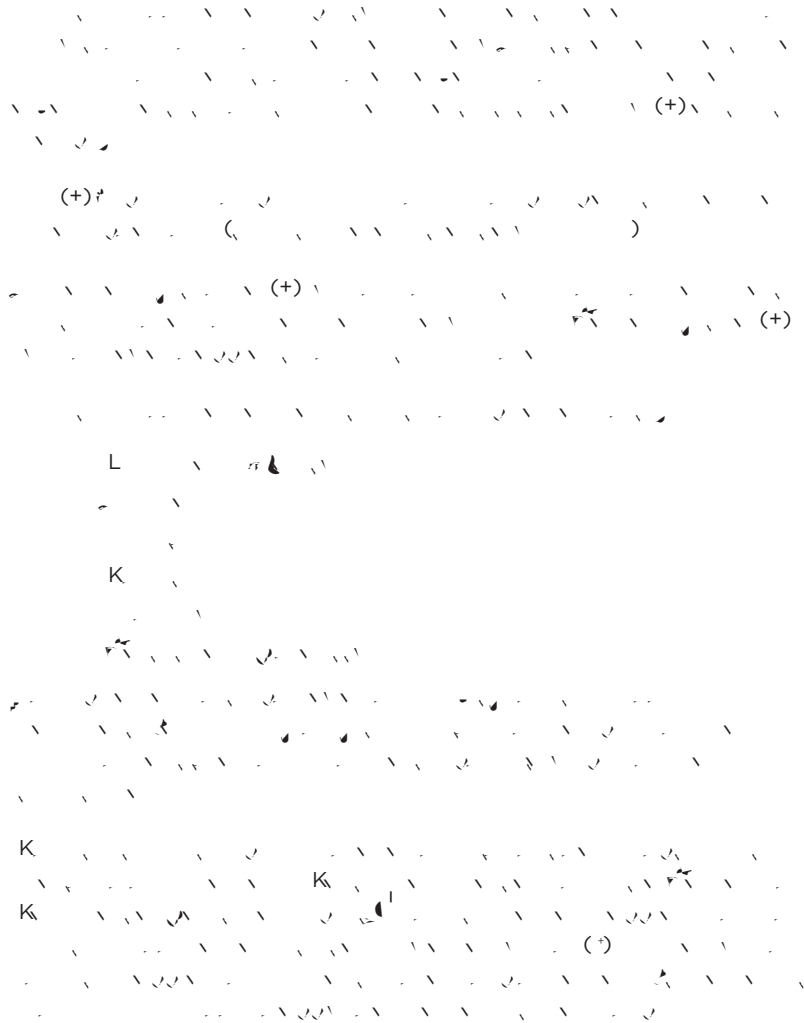
8.SP

**Investigate patterns of association in bivariate data.**





# Mathematics Standards for High School



# Mathematics | High School—Number and Quantity

## Numbers and Number Systems



### The Real Number System

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.

### Quantities

- Reason quantitatively and use units to solve problems

### The Complex Number System

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

### Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

### Mathematical Practices



## The Real Number System

N-RN

Extend the properties of exponents to rational exponents.

$$1 \quad \sqrt[3]{27} = 27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3^{\frac{3}{3}} = 3^1 = 3$$







# Algebra Overview

## Seeing Structure in Expressions

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

## Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

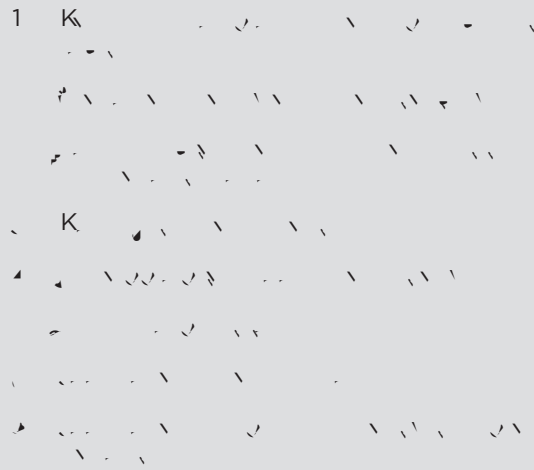
## Creating Equations

- Create equations that describe numbers or relationships

## Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

## Mathematical Practices







**Rewrite rational expressions**

$$\frac{x^2 + 5x + 6}{x^2 - 4} = \frac{(x+2)(x+3)}{(x-2)(x+2)} = \frac{x+3}{x-2}$$

Creating Equations<sup>+</sup>

A-CED

**Create equations that describe numbers or relationships**

$$1 \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{z} \quad \text{where } x, y, z \text{ are real numbers and } x, y, z \neq 0$$

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z} \quad \text{where } x, y, z \text{ are real numbers and } x, y, z \neq 0$$

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z} \quad \text{where } x, y, z \text{ are real numbers and } x, y, z \neq 0$$

$$\begin{aligned} & \text{Graph the system of linear inequalities.} \\ & y < -\frac{1}{2}x + 3 \\ & y > \frac{1}{3}x - 2 \end{aligned}$$

### Represent and solve equations and inequalities graphically

$$\begin{aligned} & \text{Graph the system of linear inequalities.} \\ & y < -\frac{1}{2}x + 3 \\ & y > \frac{1}{3}x - 2 \end{aligned}$$

## Mathematics | High School—Functions



### Interpreting Functions

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

### Building Functions

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

### Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

### Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

## Interpreting Functions

F-IF

## Understand the concept of a function and use function notation

1. Understand that a function assigns to each element  $x$  in a set  $A$  exactly one element  $y$  in a set  $B$ . Understand that the graph of a function is the set of ordered pairs  $(x, y)$  that satisfy the function.

## Building Functions

F-BF

## Build a function that models a relationship between two quantities

1. Given two quantities that change in a linear relationship, write a function that models the relationship. Be able to graph the function and interpret the slope and y-intercept in terms of the quantities. For example, if a car is traveling at a constant rate of 30 miles per hour, the total distance  $d$  traveled in time  $t$  is given by  $d = 30t$ .

## Build new functions from existing functions

1. Given a function  $y = f(x)$ , write a new function  $Y = F(x)$  that represents a transformation of  $f(x)$ . For example,  $Y = f(x) + k$  represents a vertical shift by  $k$  units. For  $f(x) = x^2$ ,  $Y = f(x) + 4$  represents a vertical shift by 4 units. For  $f(x) = x^2$ ,  $Y = f(x) - 3$  represents a vertical shift by 3 units down. For  $f(x) = x^2$ ,  $Y = f(x) + 2$  represents a vertical shift by 2 units up. For  $f(x) = x^2$ ,  $Y = f(x) - 1$  represents a vertical shift by 1 unit down.

Linear, Quadratic, and Exponential Models<sup>+</sup>

F-LE

## Construct and compare linear, quadratic, and exponential models and solve problems

1. Construct a linear, quadratic, or exponential model from a description of a situation. Interpret the parameters of the model in terms of the situation. For example, a linear model  $y = mx + b$  represents a situation with a constant rate of change  $m$  and an initial value  $b$ . A quadratic model  $y = ax^2 + bx + c$  represents a situation with a constant acceleration  $a$  and an initial velocity  $b$  and an initial position  $c$ . An exponential model  $y = a(b)^x$  represents a situation with a constant growth rate  $b$  and an initial value  $a$ .

For example, the function  $f(x) = 4x + 12$  represents the total amount of money that a family spends on a trip to the movies as a function of the number of movies watched. In this function, the input represents the number of movies watched, and the output represents the total amount of money spent. The graph of the function is shown below.



# Mathematics | High School—Modeling

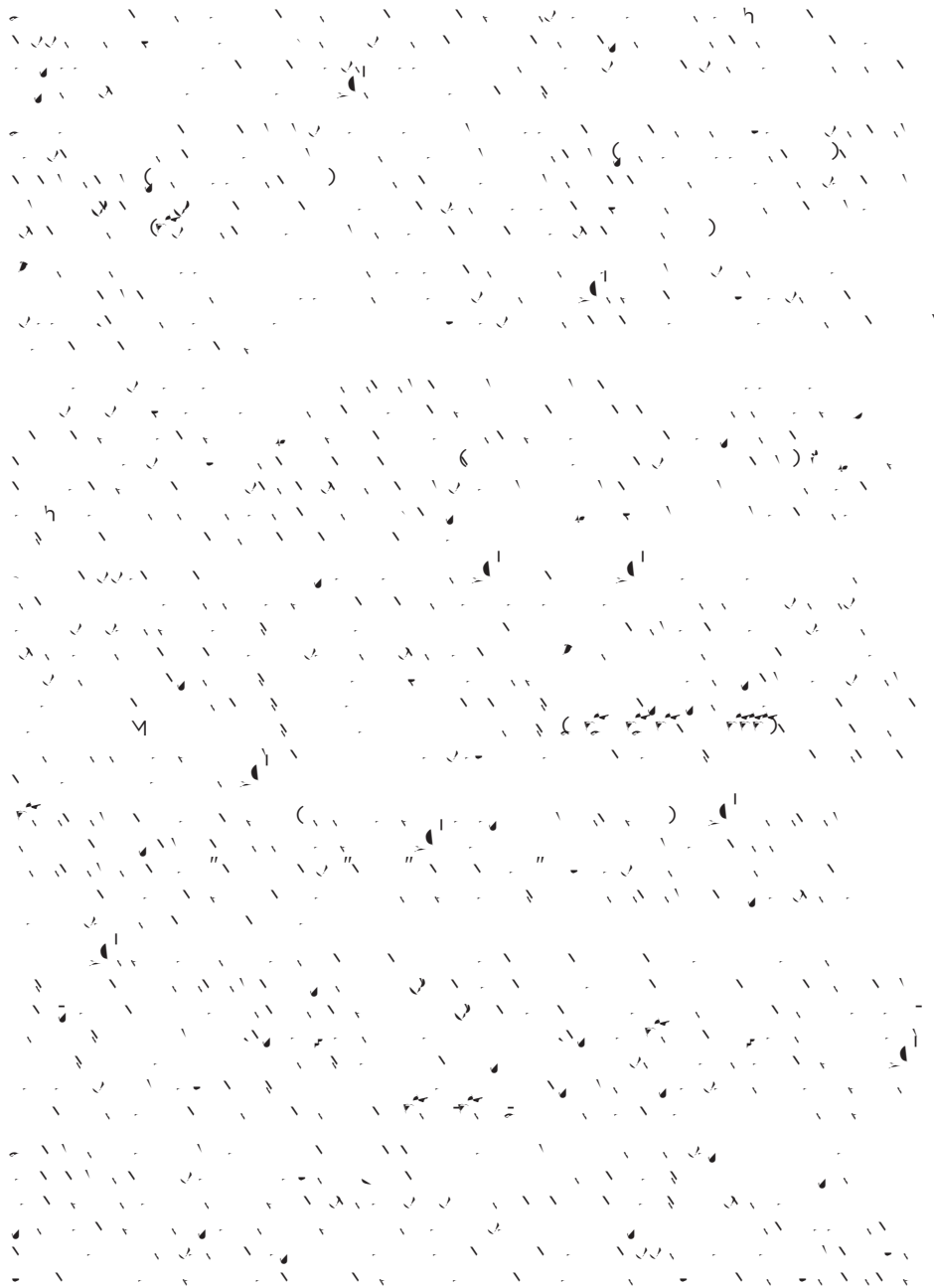




$(c)$   $\frac{1}{2}$  of  $\frac{1}{3}$  is  $\frac{1}{6}$ .  $\frac{1}{2}$  of  $\frac{2}{3}$  is  $\frac{1}{3}$ .  $\frac{1}{2}$  of  $\frac{3}{4}$  is  $\frac{3}{8}$ .  $\frac{1}{2}$  of  $\frac{4}{5}$  is  $\frac{2}{5}$ .  $\frac{1}{2}$  of  $\frac{5}{6}$  is  $\frac{5}{12}$ .  $\frac{1}{2}$  of  $\frac{6}{7}$  is  $\frac{3}{7}$ .  $\frac{1}{2}$  of  $\frac{7}{8}$  is  $\frac{7}{16}$ .  $\frac{1}{2}$  of  $\frac{8}{9}$  is  $\frac{4}{9}$ .  $\frac{1}{2}$  of  $\frac{9}{10}$  is  $\frac{9}{20}$ .  $\frac{1}{2}$  of  $\frac{10}{11}$  is  $\frac{5}{11}$ .  $\frac{1}{2}$  of  $\frac{11}{12}$  is  $\frac{11}{24}$ .  $\frac{1}{2}$  of  $\frac{12}{13}$  is  $\frac{6}{13}$ .  $\frac{1}{2}$  of  $\frac{13}{14}$  is  $\frac{13}{28}$ .  $\frac{1}{2}$  of  $\frac{14}{15}$  is  $\frac{7}{15}$ .  $\frac{1}{2}$  of  $\frac{15}{16}$  is  $\frac{15}{32}$ .  $\frac{1}{2}$  of  $\frac{16}{17}$  is  $\frac{8}{17}$ .  $\frac{1}{2}$  of  $\frac{17}{18}$  is  $\frac{17}{36}$ .  $\frac{1}{2}$  of  $\frac{18}{19}$  is  $\frac{9}{19}$ .  $\frac{1}{2}$  of  $\frac{19}{20}$  is  $\frac{19}{40}$ .  $\frac{1}{2}$  of  $\frac{20}{21}$  is  $\frac{10}{21}$ .  $\frac{1}{2}$  of  $\frac{21}{22}$  is  $\frac{21}{44}$ .  $\frac{1}{2}$  of  $\frac{22}{23}$  is  $\frac{11}{23}$ .  $\frac{1}{2}$  of  $\frac{23}{24}$  is  $\frac{23}{48}$ .  $\frac{1}{2}$  of  $\frac{24}{25}$  is  $\frac{12}{25}$ .  $\frac{1}{2}$  of  $\frac{25}{26}$  is  $\frac{25}{52}$ .  $\frac{1}{2}$  of  $\frac{26}{27}$  is  $\frac{13}{27}$ .  $\frac{1}{2}$  of  $\frac{27}{28}$  is  $\frac{27}{56}$ .  $\frac{1}{2}$  of  $\frac{28}{29}$  is  $\frac{14}{29}$ .  $\frac{1}{2}$  of  $\frac{29}{30}$  is  $\frac{29}{60}$ .  $\frac{1}{2}$  of  $\frac{30}{31}$  is  $\frac{15}{31}$ .  $\frac{1}{2}$  of  $\frac{31}{32}$  is  $\frac{31}{64}$ .  $\frac{1}{2}$  of  $\frac{32}{33}$  is  $\frac{16}{33}$ .  $\frac{1}{2}$  of  $\frac{33}{34}$  is  $\frac{33}{68}$ .  $\frac{1}{2}$  of  $\frac{34}{35}$  is  $\frac{17}{35}$ .  $\frac{1}{2}$  of  $\frac{35}{36}$  is  $\frac{35}{72}$ .  $\frac{1}{2}$  of  $\frac{36}{37}$  is  $\frac{18}{37}$ .  $\frac{1}{2}$  of  $\frac{37}{38}$  is  $\frac{37}{76}$ .  $\frac{1}{2}$  of  $\frac{38}{39}$  is  $\frac{19}{39}$ .  $\frac{1}{2}$  of  $\frac{39}{40}$  is  $\frac{39}{80}$ .  $\frac{1}{2}$  of  $\frac{40}{41}$  is  $\frac{20}{41}$ .  $\frac{1}{2}$  of  $\frac{41}{42}$  is  $\frac{41}{84}$ .  $\frac{1}{2}$  of  $\frac{42}{43}$  is  $\frac{21}{43}$ .  $\frac{1}{2}$  of  $\frac{43}{44}$  is  $\frac{43}{88}$ .  $\frac{1}{2}$  of  $\frac{44}{45}$  is  $\frac{22}{45}$ .  $\frac{1}{2}$  of  $\frac{45}{46}$  is  $\frac{45}{92}$ .  $\frac{1}{2}$  of  $\frac{46}{47}$  is  $\frac{23}{47}$ .  $\frac{1}{2}$  of  $\frac{47}{48}$  is  $\frac{47}{96}$ .  $\frac{1}{2}$  of  $\frac{48}{49}$  is  $\frac{24}{49}$ .  $\frac{1}{2}$  of  $\frac{49}{50}$  is  $\frac{49}{100}$ .  $\frac{1}{2}$  of  $\frac{50}{51}$  is  $\frac{25}{51}$ .  $\frac{1}{2}$  of  $\frac{51}{52}$  is  $\frac{51}{104}$ .  $\frac{1}{2}$  of  $\frac{52}{53}$  is  $\frac{26}{53}$ .  $\frac{1}{2}$  of  $\frac{53}{54}$  is  $\frac{53}{108}$ .  $\frac{1}{2}$  of  $\frac{54}{55}$  is  $\frac{27}{55}$ .  $\frac{1}{2}$  of  $\frac{55}{56}$  is  $\frac{55}{112}$ .  $\frac{1}{2}$  of  $\frac{56}{57}$  is  $\frac{28}{57}$ .  $\frac{1}{2}$  of  $\frac{57}{58}$  is  $\frac{57}{116}$ .  $\frac{1}{2}$  of  $\frac{58}{59}$  is  $\frac{29}{59}$ .  $\frac{1}{2}$  of  $\frac{59}{60}$  is  $\frac{59}{120}$ .  $\frac{1}{2}$  of  $\frac{60}{61}$  is  $\frac{30}{61}$ .  $\frac{1}{2}$  of  $\frac{61}{62}$  is  $\frac{61}{124}$ .  $\frac{1}{2}$  of  $\frac{62}{63}$  is  $\frac{31}{63}$ .  $\frac{1}{2}$  of  $\frac{63}{64}$  is  $\frac{63}{128}$ .  $\frac{1}{2}$  of  $\frac{64}{65}$  is  $\frac{32}{65}$ .  $\frac{1}{2}$  of  $\frac{65}{66}$  is  $\frac{65}{132}$ .  $\frac{1}{2}$  of  $\frac{66}{67}$  is  $\frac{33}{67}$ .  $\frac{1}{2}$  of  $\frac{67}{68}$  is  $\frac{67}{136}$ .  $\frac{1}{2}$  of  $\frac{68}{69}$  is  $\frac{34}{69}$ .  $\frac{1}{2}$  of  $\frac{69}{70}$  is  $\frac{69}{140}$ .  $\frac{1}{2}$  of  $\frac{70}{71}$  is  $\frac{35}{71}$ .  $\frac{1}{2}$  of  $\frac{71}{72}$  is  $\frac{71}{144}$ .  $\frac{1}{2}$  of  $\frac{72}{73}$  is  $\frac{36}{73}$ .  $\frac{1}{2}$  of  $\frac{73}{74}$  is  $\frac{73}{148}$ .  $\frac{1}{2}$  of  $\frac{74}{75}$  is  $\frac{37}{75}$ .  $\frac{1}{2}$  of  $\frac{75}{76}$  is  $\frac{75}{152}$ .  $\frac{1}{2}$  of  $\frac{76}{77}$  is  $\frac{38}{77}$ .  $\frac{1}{2}$  of  $\frac{77}{78}$  is  $\frac{77}{156}$ .  $\frac{1}{2}$  of  $\frac{78}{79}$  is  $\frac{39}{79}$ .  $\frac{1}{2}$  of  $\frac{79}{80}$  is  $\frac{79}{160}$ .  $\frac{1}{2}$  of  $\frac{80}{81}$  is  $\frac{40}{81}$ .  $\frac{1}{2}$  of  $\frac{81}{82}$  is  $\frac{81}{164}$ .  $\frac{1}{2}$  of  $\frac{82}{83}$  is  $\frac{41}{83}$ .  $\frac{1}{2}$  of  $\frac{83}{84}$  is  $\frac{83}{168}$ .  $\frac{1}{2}$  of  $\frac{84}{85}$  is  $\frac{42}{85}$ .  $\frac{1}{2}$  of  $\frac{85}{86}$  is  $\frac{85}{172}$ .  $\frac{1}{2}$  of  $\frac{86}{87}$  is  $\frac{43}{87}$ .  $\frac{1}{2}$  of  $\frac{87}{88}$  is  $\frac{87}{176}$ .  $\frac{1}{2}$  of  $\frac{88}{89}$  is  $\frac{44}{89}$ .  $\frac{1}{2}$  of  $\frac{89}{90}$  is  $\frac{89}{180}$ .  $\frac{1}{2}$  of  $\frac{90}{91}$  is  $\frac{45}{91}$ .  $\frac{1}{2}$  of  $\frac{91}{92}$  is  $\frac{91}{184}$ .  $\frac{1}{2}$  of  $\frac{92}{93}$  is  $\frac{46}{93}$ .  $\frac{1}{2}$  of  $\frac{93}{94}$  is  $\frac{93}{188}$ .  $\frac{1}{2}$  of  $\frac{94}{95}$  is  $\frac{47}{95}$ .  $\frac{1}{2}$  of  $\frac{95}{96}$  is  $\frac{95}{192}$ .  $\frac{1}{2}$  of  $\frac{96}{97}$  is  $\frac{48}{97}$ .  $\frac{1}{2}$  of  $\frac{97}{98}$  is  $\frac{97}{196}$ .  $\frac{1}{2}$  of  $\frac{98}{99}$  is  $\frac{49}{99}$ .  $\frac{1}{2}$  of  $\frac{99}{100}$  is  $\frac{99}{200}$ .

**Modeling Standards** **M**

## Mathematics | High School—Geometry



# Geometry Overview

## Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

## Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

## Circles

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

## Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

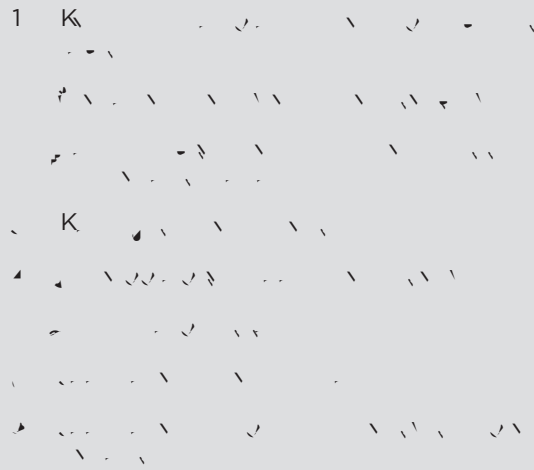
## Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

## Modeling with Geometry

- Apply geometric concepts in modeling situations

## Mathematical Practices



## Congruence

## G-CO

**Experiment with transformations in the plane****Understand congruence in terms of rigid motions****Prove geometric theorems****Make geometric constructions**

## Similarity, Right Triangles, and Trigonometry

## G-SRT

## Understand similarity in terms of similarity transformations





# Mathematics | High School—Statistics and Probability +



# Statistics and Probability Overview

## Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

## Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

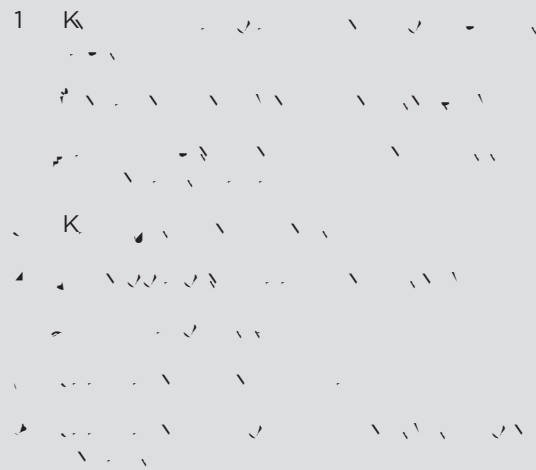
## Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

## Using Probability to Make Decisions

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

## Mathematical Practices







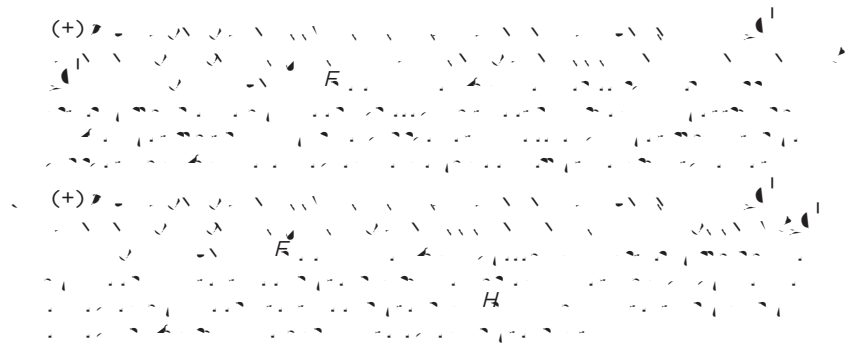


## Conditional Probability and the Rules of Probability

S-CP

**Understand independence and conditional probability and use them to interpret data**

1. Understand conditional probability and probability of independent events.
- (a) Compute conditional probabilities for events.  $P(A|B)$  is the fraction of outcomes in which  $A$  occurs, given that  $B$  has occurred.  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ .
- (b) Recognize and identify the complement of an event  $A$ , denoted  $A^c$ .
- (c) Compute probabilities of independent events.  $P(A \cap B) = P(A)P(B)$  if  $A$  and  $B$  are independent events.



**Use probability to evaluate outcomes of decisions**

4



**Addition and subtraction within 5, 10, 20, 100, or 1000.**

**Additive inverses**

**Associative property of addition**

**Associative property of multiplication.**

**Bivariate data.**

**Box plot**

**Commutative property**

**Complex fraction**  $\frac{A/B}{C/D}$ ,  $\frac{A}{B/C}$ ,  $\frac{A/B}{C}$ ,  $\frac{A}{B/C/D}$ ,  $\frac{A}{(B/C)}$

**Computation algorithm.**

**Computation strategy**

**Congruent**

**Integer.**

**Interquartile Range.**

**Line plot.**

**Mean.**

**Mean absolute deviation.**

**Median.**

**Midline.**

**Multiplication and division within 100 K.**

**Multiplicative inverses.**

**Number line diagram.**

**Percent rate of change.**

**Probability distribution.**

**Properties of operations**

**Properties of equality**

**Properties of inequality**

**Properties of operations**

**Probability.**

**Probability model.**

**Random variable.**

**Rational expression.**

**Rational number.**

**Rectilinear figure.**

**Rigid motion.**

**Repeating decimal**

**Sample space**

**Scatter plot**

**Similarity transformation**

**Tape diagram**

**Terminating decimal**

**Third quartile**

**Transitivity principle for indirect measurement**

TABLE 1. *Grade 7 Mathematics Standards*

Grade	Standard	Standard Number	Standard Description
7	Number and Operations—Fractions	1	Add, subtract, multiply, and divide positive and negative fractions and mixed numbers.
		2	Convert, multiply, and divide units within a measurement system, using conversion factors and unit rates.
		3	Use operations with fractions to solve word problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers.
Number and Operations—Real Number System	1	Understand rational numbers as extensions of the rational number system.	
	2	Understand that every rational number has a unique representation on the number line.	
	3	Understand addition, subtraction, multiplication, and division of rational numbers as extensions of the operations of addition, subtraction, multiplication, and division of rational numbers.	
Number and Operations—Integers	1	Understand that every integer has a unique representation on the number line.	
	2	Understand addition, subtraction, multiplication, and division of integers as extensions of the operations of addition, subtraction, multiplication, and division of integers.	
	3	Use operations with integers to solve word problems involving addition, subtraction, multiplication, and division of integers.	
Number and Operations—Decimals	1	Understand that every decimal has a unique representation on the number line.	
	2	Understand addition, subtraction, multiplication, and division of decimals as extensions of the operations of addition, subtraction, multiplication, and division of decimals.	
	3	Use operations with decimals to solve word problems involving addition, subtraction, multiplication, and division of decimals.	
Number and Operations—Rational Numbers	1	Understand that every rational number has a unique representation on the number line.	
	2	Understand addition, subtraction, multiplication, and division of rational numbers as extensions of the operations of addition, subtraction, multiplication, and division of rational numbers.	
	3	Use operations with rational numbers to solve word problems involving addition, subtraction, multiplication, and division of rational numbers.	
Number and Operations—Complex Numbers	1	Understand that every complex number has a unique representation on the complex plane.	
	2	Understand addition, subtraction, multiplication, and division of complex numbers as extensions of the operations of addition, subtraction, multiplication, and division of complex numbers.	
	3	Use operations with complex numbers to solve word problems involving addition, subtraction, multiplication, and division of complex numbers.	





TABLE 3

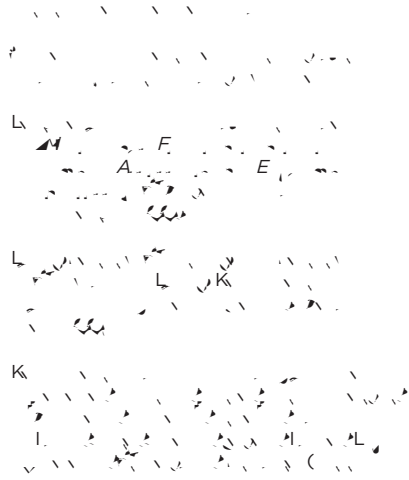
A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
G	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
E	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
G	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
E	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
D	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$

TABLE 4

R	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
D	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$

TABLE 5

A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$
A	$(-a)^n = (-1)^n a^n$	$(-a)^n = (-1)^n a^n$



$\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6} = \frac{1}{3}$   
 $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$   
 $\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$

$\frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$   
 $\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$   
 $\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$

$\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8}$   
 $\frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{3 \times 2} = \frac{1}{6}$   
 $\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$

$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$   
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$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$   
 $\frac{1}{3} \times \frac{1}{4} = \frac{1 \times 1}{3 \times 4} = \frac{1}{12}$   
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$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$   
 $\frac{1}{3} \times \frac{1}{4} = \frac{1 \times 1}{3 \times 4} = \frac{1}{12}$   
 $\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$

